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Regular Session:

Speaker: Ralph Willox (University of Tokyo, Japan)

Collaborators: Basil Grammaticos, Alfred Ramani

Title: *A peculiar nonautonomous form of the Lyness mapping*

Abstract: We examine possible integrable deautonomisations of the family of mappings commonly known as ‘the Lyness mapping’ [1–3] — N^{th} -order integrable discrete systems ($N \geq 2$) that can be generated from a one-dimensional reduction of the Hirota-Miwa equation— based on the structure of their singularities. We show that when the mappings are given in their standard form,

$$x_{n+N}x_n = a + x_{n+1} + x_{n+2} + \cdots + x_{n+N-1},$$

only the $N = 2$ case has a natural deautonomisation, whereas the mappings in their ‘derivative form’,

$$x_{n+N}(1 + x_n) = x_{n+1}(1 + x_{n+N+1}),$$

have natural, integrable, deautonomisations for all $N \geq 3$. However, the deautonomisation of the derivative form in the $N = 2$ case turns out to possess a feature we have never met before: the secular dependence on n in the coefficients of the mapping enters through two different exponential terms instead of just a single one. As a consequence, as a special subcase, the same mapping can also have an integrable nonautonomous form in which the n dependence appears in an additive fashion in the coefficients instead of the generic multiplicative dependence.

This talk is based on a paper together with Basil Grammaticos and Alfred Ramani [4].

References:

- [1] Lyness, R.C. (1942) “Mathematical Notes 1581. Cycles”, *The Mathematical Gazette* **26**, 207–207.
- [2] Lyness, R.C. (1945) “Mathematical Notes 1847. Cycles”, *The Mathematical Gazette* **29**, 231–233.
- [3] Kocic, V.L., Ladas G. and Rodrigues, I.W. (1993) “On recursive sequences”, *J. Math. Anal. Appl.* **173**,127–157.
- [4] Grammaticos, B., Ramani, A. and Willox, R. (2026) “Deautonomising the Lyness mapping”, arXiv:2603.24871 [nlin.SI].