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Title: *Application of Homogeneous Spherical Polynomials to Study the Dynamics of Perturbations of the Rossby-Haurwits Waves*

Abstract: The vorticity equation describing the motion of an unforced ideal incompressible fluid on a rotating sphere takes into account nonlinear interaction, dispersion and rotation of the sphere. The model of an ideal fluid, obeying a number of integral conservation laws, has been an important object of application of mathematical methods for the study of nonlinear hydrodynamics for decades.

The work considers meteorologically important Rossby–Haurwitz (RH) waves, which are classical solutions of the nonlinear vorticity equation for ideal flows on a rotating sphere. These waves belong to subspaces $\mathbf{H}_1 \oplus \mathbf{H}_n$ $(n \geq 2)$, where $\mathbf{H}_k$ is the subspace of homogeneous spherical polynomials of degree k. The dynamics and stability of their perturbations are analyzed. A conservation law is derived that divides arbitrary (and not just infinitesimal) perturbations of the RH wave into invariant disjoint sets $\mathbf{M}_-^n$, $\mathbf{M}_0^n$ and $\mathbf{M}_+^n$. The set $\mathbf{M}_0^n$ separating $\mathbf{M}_-^n$ and $\mathbf{M}_+^n$ contains the invariant subspace $\mathbf{H}_n$ of neutral (stable) perturbations.

A factor space and norms of perturbations are introduced, and the relationship between the enstrophy norm, the energy norm, and the factor norm of perturbations is demonstrated. A hyperbolic law between the energy and average spectral number of perturbations belonging to the sets $\mathbf{M}_-^n$ and $\mathbf{M}_+^n$ is obtained. It is shown that the energy cascade of growing perturbations of the RH wave has opposite directions in the invariant sets $\mathbf{M}_-^n$ and $\mathbf{M}_+^n$.

A geometric interpretation of the change in perturbation energy is given. Examples of the dynamics of RH wave perturbations are considered. The Lyapunov instability of any nonstationary non-zonal RH wave from $\mathbf{H}_1 \oplus \mathbf{H}_n$ $(n \geq 2)$ is proved, and the instability mechanism is explained.

Using the conservation law for perturbations, a new necessary condition for linear instability is obtained. For zonal solutions (Legendre polynomials and zonal RH waves), it complements the well-known Rayleigh–Kuo and Fjörtoft conditions. The bounds on the growth rate of unstable modes are also given, and the orthogonality of the amplitude of unstable modes to the basic RH wave is demonstrated. The results clarify the spectral structure of unstable disturbances and are useful for testing computational algorithms in numerical studies of linear stability.