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Regular Session:

Speaker: Anton Shchekkin (SISSA, INFN, and IGAP, Trieste, Italy)

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Title: *Refined Painlevé/gauge theory correspondence*

Abstract: We present bilinear tau forms of the quantum Painlevé equations and describe their solutions near critical points in terms of supersymmetric gauge theory partition functions.

We seek these solutions as Zak transforms of certain (asymptotic) series around critical points of the (quantum) Painlevé equations, by quantizing the ansatz of seminal paper [1] and subsequent [2]. We compute several leading terms of these series and fix the arising freedom of the bilinear tau forms from the Hamiltonian forms of the quantum Painlevé equations. We then identify the obtained series with the (asymptotic) expansions of partition functions of four-dimensional $\mathcal{N} = 2$ $SU(2)$ supersymmetric gauge theories with appropriate Ω -background. In this way, we refine the Painlevé/gauge theory correspondence of [1], [2].

We mainly focus on the asymptotic expansions around the irregular critical point $t = \infty$, which correspond to the strong-coupling regime of the gauge theories, including Argyres-Douglas points. On the gauge-theory side, we compute these expansions using the holomorphic anomaly equations. We also identify them with expansions of irregular conformal blocks, thereby clarifying the corresponding AGT relation.

This talk is based on [3] and its companion paper [4].

References

- [1] O. Gamayun, N. Iorgov, and O. Lisovyy, *Conformal field theory of Painlevé VI*. In: JHEP 10, 183 (2012), p. 038. arXiv: 1207.0787 [hep-th]

- [2] G. Bonelli, O. Lisovyy, K. Maruyoshi, A. Sciarappa, and A. Tanzini, *On Painlevé/gauge theory correspondence*. In: Lett. Math. Phys. 107.12 (2017), pp. 2359–2413. arXiv: 1612.06235 [hep-th].
- [3] G. Bonelli, A. Shchepochkin, A. Tanzini, *Refined Painlevé/gauge theory correspondence and quantum tau functions*. In: Annales Henri Poincaré (2025). arXiv: 2502.01499 [hep-th]
- [4] G. Bonelli, A. Shchepochkin, A. Tanzini, *Bilinear tau forms of quantum Painlevé equations and $\mathbb{C}^2/\mathbb{Z}_2$ blowup relations in SUSY gauge theories*. arXiv: 2512.25051 [math-ph]