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Title: *Exact Integration of Nonlinear PDEs via $\mathcal{C}^\infty$-Structures*

Abstract: We address the problem of finding explicit solutions to nonlinear PDEs by combining the method of differential constraints with the theory of $\mathcal{C}^\infty$-structures. The method of differential constraints [1, 2, 6] reduces a PDE to an overdetermined system whose compatibility can be studied via its Vessiot distribution. Recent applications of compatible differential constraints to nonlinear evolution equations [7] illustrate the scope of this strategy. Moreover, when this distribution is involutive on an open set and the rank equals the number of independent variables, it can be integrated by quadratures using a solvable structure [3, 4], though solvable structures remain difficult to construct in practice.

We propose replacing them by $\mathcal{C}^\infty$-structures of distributions [5], a generalization, with relaxed bracket conditions, that is directly analogous to the extension of Lie point symmetries to $\mathcal{C}^\infty$-symmetries (or λ-symmetries) in the ODE setting. Just as λ-symmetries of an ODE enable integration even when no Lie point symmetry exists, a $\mathcal{C}^\infty$-structure of a Vessiot distribution provides a systematic sequence of completely integrable Pfaffian equations whose solution yields the exact integral manifolds of the distribution, and hence explicit solutions of the original PDE.

We illustrate the method with explicit examples on nonlinear evolution equations where classical solvable structures are unavailable.

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