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Regular Session:

Speaker: Frank Nijhoff (University of Leeds, UK & Shanghai University, PR China)

Collaborators: Steven King, Cheng Zhang, Da-jun Zhang, Peter van der Kamp.

Title: *Lattice Gel'fand-Dikii mappings and the associative Yang-Baxter equation*

Abstract: The lattice Gel'fand-Dikii (GD) hierarchy was introduced in [1], and its mapping reductions, under periodic initial conditions, were derived. These families of mappings possess two Lax representations: a zero-curvature ('small') Lax pair, and a dual ('big') Lax pair. In the case of the small Lax pair a non-ultralocal classical r -matrix structure was given, which can be readily generalised to the quantum case, [2, 3]. In contrast, the r -matrix structure for the dual Lax representation remained elusive for a long time, and was given only in the special case of KdV mappings much later in [4]. The dual r -matrix has a rather curious structure, which will be elucidated in the talk, and obeys the so-called *associative Yang-Baxter equation* (a terminology coined by A. Kirillov). It will be shown that the relevant r -matrix extends to classical and quantum mappings associated with the entire GD hierarchy. We will discuss the associated algebraic structures and various generalisations. This is work partly in collaboration with Steven King, and partly with Cheng Zhang, Da-jun Zhang and Peter van der Kamp.

References

- [1] F.W. Nijhoff, V.G. Papageorgiou, H.W. Capel and G.R.W. Quispel, *The lattice Gel'fand-Dikii hierarchy*, Inverse Problems **8** (1992), no. 4, 597–622.
- [2] F.W. Nijhoff and H.W. Capel, *Integrable quantum mappings and non-ultralocal Yang-Baxter structures*, Phys. Lett. **A163** (1992) 49–56.

- [3] F.W. Nijhoff and H.W. Capel, *Integrability and fusion algebra for quantum mappings*, J. Phys. A: Math. Gen. **26** (1993) 6385–6407.
- [4] S.D. King, *Lagrangian Multiform Structures, Discrete Systems and Quantisation*, Leeds University PhD thesis, August 2017.