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Title: *Pencils of Novikov algebras of Stäckel type and soliton hierarchies*

Abstract: There exist various ways of constructing soliton hierarchies from appropriate algebraic structures. For example, in [10] the authors used loop algebras and r -matrix theory to produce compatible Poisson brackets leading to cKdV and cHD hierarchies. In [7] Frobenius algebras were applied to multi-component third-order local Poisson structures. In the article [13], the authors performed the construction of $(1+1)$ -dimensional integrable bi-Hamiltonian systems associated with Novikov algebras. The obtained systems were multi-component generalizations of the Camassa-Holm equation [8] that can be interpreted as Euler equations on the respective centrally extended Lie algebras. A similar approach for constructing multi-component soliton hierarchies, specifically Harry Dym and Hunter-Saxton, based on Frobenius triple, has been presented in [12].

The homogeneous first-order Hamiltonian operators [11, 4], which are a special case of the Dubrovin-Novikov operators of hydrodynamic type [9], have a very natural underlying algebraic structure. The conditions for a homogeneous operator

$$\Pi^{ij} = \frac{1}{2}(b_k^{ij} + b_k^{ji})u^k \frac{d}{dx} + \frac{1}{2}b_k^{ij}u_x^k, \quad (1)$$

to be Hamiltonian are such that the b_k^{ij} are the structure constants of a Novikov algebra [4]. Moreover, these operators can be defined through Lie-Poisson structures associated with the so-called translationally invariant Lie algebras, which are in one-to-one correspondence with Novikov algebras.

In this talk we show a way of constructing evolutionary soliton hierarchies from pencils of Novikov algebras of Stäckel type. We begin by defining a special class of associative Novikov algebras, which we call Novikov algebras of Stäckel type, as they are associated with classical Stäckel metrics in Viète coordinates. We obtain sufficient conditions for pencils of these algebras so that the corresponding Dubrovin-Novikov Hamiltonian operators can be centrally extended, producing sets of pairwise compatible Poisson operators. These operators lead to coupled Korteweg-de Vries (cKdV) and coupled Harry Dym (cHD) hierarchies [1, 2, 3, 5] as well as to a triangular cKdV hierarchy and a triangular cHD hierarchy.

The content of this talk can be found in [6]

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