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A Talk at the 2nd ISNMP Conference

Bad Ems, 28 June to 4 July 2026

Regular Session:

Speaker: Roman Kozlov (Norwegian School of Economics, Norway)

Title: *Conservation properties of mean field games equations*

Abstract: The Mean Field Games (MFG) theory provides a mathematical framework for understanding the behavior of large populations of interacting agents, where the entire population influences each agent's behavior. The PDE approach to MFG theory is a rapidly developing field of research that has its roots in the seminal work of J.-M. Lasry and P.-L. Lions [1]. Another approach to the development of MFG theory was suggested by M. Huang, R. Malhamé, and P. Caines [2].

The system of mean field games equations consists of two partial differential equations: the Hamilton-Jacobi-Bellman equation for the value function and the forward Kolmogorov equation for the probability density. For separable Hamiltonians, this system has a variational structure: its equations are Euler-Lagrange equations for some Lagrangian functions. Therefore, one can use the Noether theorem to derive the conservation laws using variational and divergence symmetries.

The presentation considers separable, state-independent Hamiltonians in one-dimensional state space [3, 4, 5]. First, the most general form of the mean field games system is examined for symmetries and conservation laws. Then, particular cases of the system that lead to additional symmetries and conservation laws are identified.

References

[1] J.-M. Lasry and P.-L. Lions, Mean field games, *Japanese Journal of Mathematics*, **2**(1), 229–260, 2007

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[3] R. Kozlov, Conservation laws of mean field games equations, *Communications in Nonlinear Science and Numerical Simulation*, **147**, 108796, 2025.

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