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Open Problems Session:

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Title: *The alternated composition of $N = 2p$ differential operators $w_j(x)\,\partial_x^p$ yields the weights' Wronskian with which constant?*

Abstract: We study strongly homotopy deformations of Lie algebras realised by vector fields on the (complex) line, e.g., polynomial realisations of finite-dimensional Lie algebras like $\mathfrak{sl}(2)$ or Laurent polynomial realisations of infinite-dimensional Lie algebras like the Witt algebra of holomorphic vector fields on $\mathbb{C}$. The Lie bracket of two vector fields is a vector field; its coefficient is the Wronskian of the two old coefficients. In the course of homotopy deformation, vector fields extend to higher-order differential operators, and the binary bracket acquires the tail of multi-linear antisymmetric brackets of many arguments. The alternated composition of $N = 2p$ differential operators $w_j(x)\,\partial_x^p$ of strict order $p \in \mathbb{N}_{\geqslant 1}$ on the line $\mathbb{R} \ni x$ is again a differential operator of strict order p; its coefficient is the constant $\boldsymbol{c}(p)$, depending only on the arity N, times the Wronskian determinant of the originally taken coefficients $w_1, \ldots, w_N$. The case $p = 1$ of the Lie bracket for two vector fields fixes $\boldsymbol{c}(1) = 1$. When $p = 2$, finding $\boldsymbol{c}(2) = 2$ is easy; next, $\boldsymbol{c}(3) = 90$. The problem is to know $\boldsymbol{c}(p \geqslant 4)$.

In arXiv:2605.11137 [math.CO] (joint work in progress with K. C. Shah) we express the formula of $\boldsymbol{c}(p)$ in terms of the sum with signs over the subset ($\subsetneq \mathbb{S}_{2p}$) of 'late-growing' permutations, thus reaching the exact values $\boldsymbol{c}(p = 4) = 586\,656$, $\boldsymbol{c}(p = 5) = 1.9151\ldots \cdot 10^{12}$, …, $\boldsymbol{c}(p = 13) = 8.3963\ldots \cdot 10^{197}$; the integer sequence $\boldsymbol{c}(p)$ seems to be new.

Problem. Is it true that the constants never vanish, $\boldsymbol{c}(p) \neq 0$, and $\boldsymbol{c}(p) > 0$ for all $p \in \mathbb{N}_{\geqslant 1}$?

- What is the (asymptotic) growth law for the integer sequence $\boldsymbol{c}(p)$ as $p \to +\infty$?

  To facilitate the answer, we ask:

- What is the law of prime decomposition for $\boldsymbol{c}(p)$?

We observe that each known value $\boldsymbol{c}(p)$ is the product of a nucleous of small primes ($\leqslant p$) in high powers, times a prime from the range $p \ldots 2p$, times a few very big primes.

- What is the law of growth for the *least prime* $\geqslant N$, and for the *largest prime* in the decomposition of $\boldsymbol{c}(p)$ as $p \to +\infty$?

The caveat $\boldsymbol{c}(p) \not\equiv 1$ could mean the following in Theoretical Physics: whenever one attempts an L_∞-deformation of the Witt algebra of holomorphic vector fields on the complex plane $\mathbb{C}$ or of a finite-dimensional Lie algebra realised by vector fields, and whenever the deformation itself is realised by higher-order differential operators, the *norms* of the expansion coefficients $w_j(x)$ must cumulatively decay faster than the pre-factor $\boldsymbol{c}(p)$. Otherwise, the deformation becomes formal, as the coefficients blow up under iteration of the bracket. In summary, does any "physical sense" predict the decay of the coefficients within the homotopy deformation tails, or does it make the L_∞-deformations of the Witt algebra in CFT taboo ?