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Regular Session:

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Title: *Wronskians over multidimension: From $\mathfrak{sl}(2)$ to (in)finite-dimensional polynomial homotopy Lie algebras*

Abstract: By commuting three vector fields on the line $\mathbb{R} \ni x$ with monomial coefficients 1, $-2x$, and $-x^2$, we realise the Lie algebra $\mathfrak{sl}(2)$ in its Chevalley basis; the bracket acts on the coefficients as the Wronskian determinant. Let us extend this model to a class of polynomial homotopy Lie algebras in which the N-ary brackets are given by the Wronskian determinants over multidimension; the generalised Vandermonde determinants then express the structure constants.

§1. The alternated composition of $N = 2p$ differential operators $w_j(x)\,\partial_x^p$ of strict order p on the line $\mathbb{R} \ni x$ is again a differential operator of strict order p; its coefficient is the constant $c(p)$ times the Wronskian determinant of the coefficients $w_1, \ldots, w_N$. At $p = 1$, the $\mathfrak{sl}(2)$ case fixes $c(1) = 1$; easy is $c(2) = 2$, then $c(3) = 90$. In a recent joint work with K. C. Shah, we reach the exact values $c(p = 4) = 586\,656$, $c(p = 5) \approx 1.9 \cdot 10^{12}$, and $c(p = 6) \approx 7.9 \cdot 10^{21}$. The positive integer sequence $c(p)$ seems to be new; to know $c(p \geqslant 7)$ is an open problem.

§2. Deform the binary Lie bracket to a formal sum of Wronskians with purely even $(N = 2p)$ or arbitrary $(N \in \mathbb{N}_{\geqslant 2})$ arities, see `arXiv:2510.02145` [math.RA]. Not only does the full bracket Δ satisfy the Jacobi identity $\Delta[\Delta] = 0$ for homotopy Lie algebra, but for every pair of arities $\ell, m \geqslant 2$ the respective (ℓ, m)-term in the identity vanishes separately. Over base dimension $d = 1$, we spot an infinite sequence of *finite*-dimensional polynomial homotopy Lie algebras starting at $\mathfrak{sl}(2)$ and with the Wronskians as the brackets; all the structure constants, unless zero due to repetitions, equal ± 1 in a suitable basis.

§3. Let the base dimension $d \geqslant 1$ be arbitrary: $\mathbb{R}^d \ni (x^1, \ldots, x^d)$. We proved in `arXiv:math.RA/04110185` that the complete generalised Wronskians – involving all the

derivatives up to a given differential order $k \geqslant 1$ – still satisfy the table of Jacobi identities for strong homotopy Lie algebras. The arity $N = \binom{d+k}{d}$ of such brackets grows with dimension d and order k but the steps, as $k \mapsto k+1$, grow as well: over $d > 1$ the gaps get larger and larger. In a recent work `arXiv:2511.03848` [math.RA] we prove that by allowing the multivariate Wronskians be *incomplete* in their top differential order $k > 1$, we do preserve all the SH-Lie Jacobi identities.

§4. For complete Wronskians of orders $k \geqslant 1$ over (multi)dimension $d \geqslant 1$ as the brackets, in a work in progress (joint with M. G. Ķēniņš) we exhaustively describe all the *finite*-dimensional polynomial N-ary SH-Lie algebra generalisations of $\mathfrak{sl}(2)$; we express their structure constants in terms of the multivariate Vandermonde determinants. Relaxing the finite-dimensionality assumption and taking the (Laurent-) monomials in d variables for the generators, we obtain multivariate analogues of the Witt algebra from CFT.