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Regular Session:

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Title: *Advances in the formal diagonalisation of systems of evolution equations*

Abstract: Integrability of multi-component evolution systems

$$u_t^i = \phi^i(\mathbf{u}, \mathbf{u}_1, \dots, \mathbf{u}_n), \quad i = 1, \dots, m$$

(where $\mathbf{u} = (u^1, \dots, u^m)$ and $\mathbf{u}_i = \partial^i \mathbf{u}/\partial x^i$) is defined, under the symmetry approach, as the existence of formal recursion $\mathbf{R}$ and symplectic $\mathbf{S}$ operators for the system. These objects are matrix pseudo-differential series satisfying the equations

$$\mathbf{R}_t = [\mathbf{\Phi}_*, \mathbf{R}],$$
$$\mathbf{S}_t + \mathbf{S}\,\mathbf{\Phi}_* + \mathbf{\Phi}_*^+\,\mathbf{S} = 0.$$

where $\mathbf{\Phi}_*$ denotes the Fréchet derivative of $\mathbf{\Phi} = (\phi^1, \dots, \phi^m)$.

Systems that satisfy some nondegeneracy condition are known to be formally diagonalisable [1] via a gauge transformation that diagonalises the Fréchet derivative $\mathbf{\Phi}_* = \mathrm{diag}(\phi_1, \dots, \phi_m)$, simplifying the equations for recursion and symplectic operators. For example, the equation for a recursion operator splits into m scalar equations $(R_i)_t = [\phi_i, R_i]$ and the well developed theory of integrable scalar evolution equations can be applied without further ado.

We will explore in this talk the possibility of diagonalising some degenerate systems, expanding the class of systems that can be studied under the symmetry approach, including examples of significant physical content. As an application, we will deduce explicit integrability conditions and produce a partial classification of integrable systems of the form

$$u_t = v_1,$$
$$v_t = u_3 + f(u, u_1, u_2, v, v_1).$$

References

[1] Mikhailov A V, Shabat A B and Sokolov V V, Symmetry Approach to Classification of Integrable Equations, *What is integrability?* Ed. V.E. Zakharov, Springer Series in Nonlinear Dynamics, Springer-Verlag, 115–184, 1991.