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Regular Session:

Speaker: M. Gorgone (University of Messina, Messina, Italy)

Collaborators: F. Oliveri

Title: *Substitution principles in ideal gas dynamics and magneto gas dynamics derived as conditional symmetries*

Abstract: In this presentation, some classical results in ideal gas dynamics and ideal magneto gas dynamics, known as *substitution principles* [1, 2, 3, 4, 5], are considered. We observe that the transformations yielding substituted flows map solutions into solutions, so they correspond to symmetries of the governing equations. We will prove that methods of Lie group analysis of differential equations allow one to derive from infinitesimal considerations the transformations involved in the substitution principles, for both steady and unsteady equations. Remarkably, suitably using Lie group methods, we are able to obtain some generalizations. The theoretical framework we use is that of conditional symmetries, where we look for the symmetries of a system of differential equations supplemented by some side conditions.

References

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