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A Talk at the 2nd ISNMP Conference

Bad Ems, 28 June to 4 July 2026

Regular Session:

Speaker: M.L. Gandarias (University of Cadiz, Cadiz, Spain)

Collaborators: S.C. Anco, A.P. Márquez, T.M. Garrido

Title: *A dissipative Westervelt's equation: symmetry analysis and hidden variational structure*

Abstract: Propagation of sound waves in a compressible medium has several important applications where nonlinear and dissipative effects are relevant. Examples are parametric arrays in water and in air, under water imaging, musical acoustics of brass instruments, sonochemistry, quality control and characterization of materials, and bio-medical devices. Especially significant is ultra-sound imaging in human tissue. A simple mathematical 1D model is given by a dissipative version of Westervelt's equation describing the pressure fluctuation. Symmetries and conservation laws are intrinsic, fundamental aspects of wave equations. Their existence is not precluded by dissipative and nonlinear effects. The present work is devoted to illustrating some of these developments for the dissipative Westervelt equation:

- Lie point symmetries of the dissipative Westervelt equation,
- conservation laws of the dissipative Westervelt equation,
- construction of the potential system,
- potential Lie point symmetries,
- potential conservation laws,
- variational structure,
- a recursion operator,
- higher–order symmetries and higher-order conservation laws.

References

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Acknowledgements: The Authors acknowledge financial support received from the research group FQM-201 of *Junta de Andalucía*.