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Student Session:

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Title: *General parametric solution of a third-order system of ODEs arising in classification of integrable systems*

Abstract: In this talk, we present the general solution to a system of two third-order ordinary differential equations (ODEs) arising from the classification problem of diagonalizable hydrodynamic chains. The Lie symmetry algebra of the system is proved to be four-dimensional and hence complete integrability by quadratures is not assured. Nevertheless, the Lie algebra permits a reduction to a non-linear, non-autonomous system of two first-order equations. Determination of symmetries of the reduced system seems to be difficult. In fact, the software Maple does not provide any symmetries for it. However, the reduced system can be transformed into a second-order ODE that admits exactly one Lie point symmetry. This underlying symmetry enables us to decouple the reduced system into an Abel equation and an auxiliary first-order ODE, the latter being solvable by quadratures once the Abel equation is integrated. The integration of the Abel equation is achieved by determining a rational first integral. Whereas theoretically the first integral is enough to solve the auxiliary equation in practice this cannot be effectively achieved due to the rather involved expression of the first integral. Alternatively, the first integral can be used to define an irreducible algebraic curve of genus zero. Exploiting this geometric property, we construct a parametric representation of the solution, allowing us to explicitly integrate the auxiliary equation. Consequently, we obtain the general solution of the reduced system and, after four subsequent quadratures, recover the exact general solution of the original third-order system in parametric form