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Student Session:

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Title: *Exact traveling waves for a damped wave equation*

Abstract: In this talk, we address the determination of exact traveling wave solutions for a strongly damped wave equation with a combined power-type nonlinearity given by

$$u_{tt} - u_{xx} + \gamma u^{m-1} u_t = \mu u - \sigma u^q, \quad q \neq 1.$$

Strongly damped wave equations have attracted much attention due to their applications in different physical fields as modeling viscoelastic fluid flows [1] and heat conduction [2]. This equation serves as a generalized damped Landau-Ginzburg-Higgs model, describing superconductivity and drift cyclotron waves in centrifugally inhomogeneous plasmas [3]. By applying the traveling wave transformation $y = x - ct$ and redefining the parameters, the equation reduces to the second-order ordinary differential equation

$$w_2 - A_1 w^{m-1} w_1 = A_2 w + A_3 w^q. \tag{1}$$

This second-order equation is studied by means of the λ-symmetry-based integration method. The notion of λ-symmetry was introduced in [4] as a generalization of the classical concept of Lie symmetry [5]. Since there, λ-symmetries have been applied in the literature to obtain exact solutions of ODEs even if they do not admit enough symmetries to guarantee integration by quadratures [6, 7].

Three different families of equations of the form (1) admitting λ-symmetries are identified, and the λ-symmetry-based integration method is applied to determine solutions to such equations. Two of them are solved for specific values of the parameter m, whereas

the other one is fully integrated. Once the families have been integrated, the corresponding traveling waves are represented for specific parameter values to illustrate particular solutions, such as kink or antikink waves, oscillatory waves, or dipole solitons.

Finally, the results obtained via the λ-symmetry method are compared with existing approaches appearing in the literature, such as the classical Lie symmetry procedure [5, 8] and the point adjoint-symmetry method [8].

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