LECTURES IN GEOMETRY OF DIFFERENTIAL EQUATIONS. CLASSICAL THEORY

(PRELIMINARY ABSTRACT)

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The aim of this book is to provide a sample how the classical geometry of (non)linear partial differential equations (PDEs) can be taught within a one-semester advanced course, covering the following topics:

- jet bundles and equations as differential-algebraic varieties;
- symmetries and invariant solutions of PDEs;
- reconstruction of conservation laws from their generating functions (or 'cosymmetries');
- Noether's First and Second Theorems for the Euler–Lagrange equations such as the Maxwell and Yang–Mills gauge systems.

This course is arranged into 16 lectures, 8 tutorials, 4 sets of homework assignments, and 2 (re)exams. The four blocks of Lecture Notes are supplemented with a bonus about recursion operators for (co)symmetry propagation.

Every lecture is accompanied by a collection of exercises, the difficulty of which ranges from basic comprehension and practice techniques to larger calculations over prolonged differential equations and ultimately, to small research problems that could potentially lead to supervised (half-)semester projects.

The book is based on the author's experience of teaching courses in the geometry of PDEs to Master's and PhD students in Utrecht (2009), Groningen (four times in 2012–2021), and within the national MasterMath programme (2018/19) to graduate students from several universities; that MasterMath course took place in VU Amsterdam.

The choice of material and execrises – as presented in these Lecture Notes – results from the author's experiences in supervising many students at different levels: from 2nd year Honours projects and Bachelor 3rd or 4th year level diploma works to Master theses and PhD dissertations.

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The book outlines the pace and ordering of a necessary minimal set of topics recommended to students who are interested in the geometry of differential equations. The lecturer can embed examples and exercises into regular instruction classes or, alternatively, choose to offer tutorials separately: these can be trusted to a teaching assistant (TA). There can also be demo-sessions showing practical calculations of structures over PDEs: mainly, their symmetries and conservation laws.

This book does *not* pursue the goal of an encyclopedia of the equations that are and of their known properties, nor is there an exhaustive list of literature references reciting all of the main directions and results by all the schools during all the stages of fashion since the KdV-boom in 1967. These Lecture Notes do also not introduce the history of the subject, pioneered by Sophus Lie in the 1870–80s.

Hamiltonian formalism for KdV-type equations, the geometry of iterated variations in the BRST/BV-quantisation of gauge systems, and the unifying view on the former and latter and on the zero-curvature representations (ZCR), or Lax pairs – through homological evolutionary vector fields and variational Lie algebroids – will be the introduced and studied in Part II as Supplementary Chapters, naturally beyond the frames of a basic, first-acquaintance course in the classical geometry of PDEs.

The early version of these notes, comprising Parts I and II with cross-references between the chapters and sharing a sequence of exercise problems, was drafted in the Spring semester 2012. 'The twelve lectures in the (non)commutative geometry of differential equations' are available on-line as IHES/M/12/13 from the IHÉS preprint series (Bures-sur-Yvette, 2012). The geometry of (iterated) variations in the Batalin–Vilkovisky (BV) formalism, corresponding to Part II of 'The 12 Lectures', is addressed in more detail in the namesake publication (2013, see `arXiv:1312.1262` [math-ph]). In the full generality of Kontsevich's formal noncommutative symplectic supergeometry, the calculus of multivectors on noncommutative jet spaces – i.e. the geometry of iterated variations, the BV-Laplacian, Schouten–Nijenhuis bracket, and Hamiltonian formalism – are explained in the article of the same name (2018, see `arXiv:1210.0726` [math.DG]); accessible to advanced students, it is suitable for a reading course.