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Student Session:

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Title: *Lie Symmetries, Reductions and Exact Solutions of a* $(2+1)$*-Dimensional Boussinesq Equation with Arbitrary Nonlinearity*

Abstract: In this talk, we investigate Lie symmetries of the $(2+1)$-dimensional Boussinesq equation, which was proposed to model the propagation of gravity waves on the water surface, with particular emphasis on the head-on collision of oblique waves. By considering the nonlinearity as an arbitrary function $f(u)$ rather than restricting it to a fixed polynomial structure, the model becomes theoretically broader and structurally more flexible. From a mathematical point of view, determining the Lie symmetry structure of the generalized equation is essential for understanding its reduction mechanisms and invariant properties. Accordingly, we establish a Lie symmetry classification with respect to the admissible forms of $f(u)$.

The obtained infinitesimal generators determine the corresponding Lie symmetry algebra and allow reductions of the governing partial differential equation to lower-dimensional equations. Making use of the optimal system of two-dimensional subalgebras of the symmetry algebra, we obtain reductions of the equation to ODEs for each of the canonical forms of $f(u)$. In particular, one of these reductions leads to a traveling-wave form, through which the equation is reduced to an ordinary differential equation. This provides a link between the symmetry structure of the equation and the traveling-wave analysis.

For the exact solution analysis, the nonlinear form $f(u) = \alpha u^2 + \beta u^3$ is considered. Under the associated traveling-wave reduction, exact solutions are obtained in terms of Jacobi elliptic functions. In certain limiting cases, these solutions reduce to hyperbolic and trigonometric wave forms, indicating the existence of localized, singular, and periodic traveling-wave structures.

Finally, the reduced ordinary differential equation is rewritten as a planar dynamical system in order to investigate the stability and qualitative behavior of the traveling-wave solutions. The corresponding phase portraits are compared with the analytical wave profiles and are shown to clarify the occurrence of different solution types under various parameter regimes.