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Regular Session:

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Title: *A Mathematica package for the symmetry approach to integrability*

Abstract:

We are interested on the symmetry approach to integrability. In the case of multi-component evolution systems, $u^i_t = \phi^i(\mathbf{u}, \mathbf{u}_1, \ldots, \mathbf{u}_n)$ $(i = 1, \ldots, m,\ \mathbf{u} = (u^1, \ldots, u^m),$ $\mathbf{u}_i = \partial^i \mathbf{u}/\partial x^i)$, integrability under the symmetry approach means the existence of formal recursion $\mathbf{R}$ and symplectic operators $\mathbf{S}$ such that

$$\mathbf{R}_t = [\mathbf{\Phi}_*, \mathbf{R}]\,, \quad \mathbf{S}_t + \mathbf{S}\mathbf{\Phi}_* + \mathbf{\Phi}_*^+\mathbf{S} = 0\,, \quad \mathbf{\Phi} = (\phi^1, \ldots, \phi^m)\,.$$

$\mathbf{\Phi}_*$ is the Fréchet derivative of $\mathbf{\Phi}$, a matrix of differential operators

$$\mathbf{\Phi}_*[i,j] = \sum_{l=0}^{n} \frac{\partial \phi^i}{\partial u^j_l} D^l\,, \qquad D = \frac{d}{dx}\,.$$

However, formal recursion $\mathbf{R}$ and symplectic $\mathbf{S}$ operators are matrix pseudo-differential series,

$$\mathbf{R} = \sum_{l=-r}^{\infty} \mathbf{R}_l D^{-l}\,, \qquad \mathbf{S} = \sum_{l=-s}^{\infty} \mathbf{S}_l D^{-l}\,.$$

We are implementing the algebra of differential and pseudo-differential operators on a Mathematica package on the jet space, so that integrability conditions can be recovered beyond the limits of manual computation. The goal is also not restricting the functions to polynomials but allowing for general functions.

The package is coded as a set of substitution rules acting on passive Mathematica operators (**pd** for partial derivative, **td** for total derivative),... Some of these rules can be applied via instructions like **applyTD** or **applyTD3**. We have a semi-automatic workflow, so that we can leave most of the computation to automatic algorithms. And at the same time complex manual solutions can be added when needed.

References

[1] Mikhailov A V, Shabat A B and Sokolov V V, Symmetry Approach to Classification of Integrable Equations, *What is integrability?* Ed. V.E. Zakharov, Springer Series in Nonlinear Dynamics, Springer-Verlag, 115–184, 1991.